Bach's Well Tempered Tuning

John Charles Francis
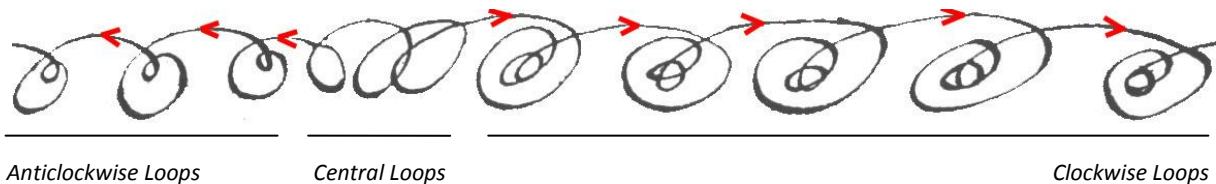
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Analysis

Tuning information abstracted from the cover sheet of J. S. Bach's Well Tempered Clavier (1722) shows three central loops with three single-knotted anticlockwise loops to the left and five double-knotted clockwise loops to the right.

Anticlockwise Loops   *Central Loops*   *Clockwise Loops*
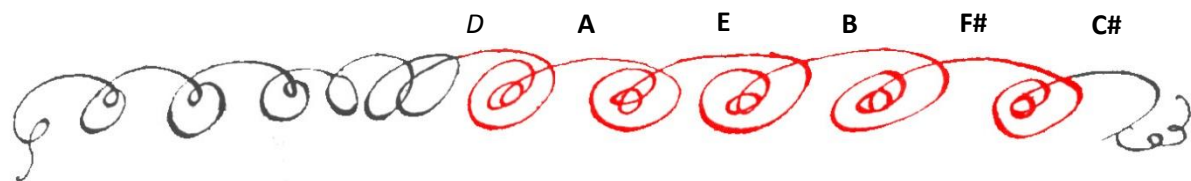
The central loops represent the tempering of fifths on the subdominant, tonic and dominant of C.
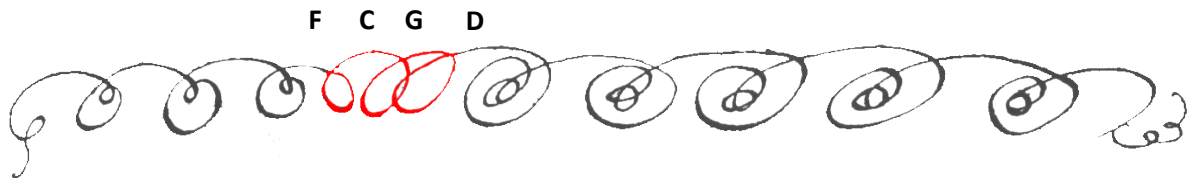
The anticlockwise loops represent the tempering of fifths going towards the flats.
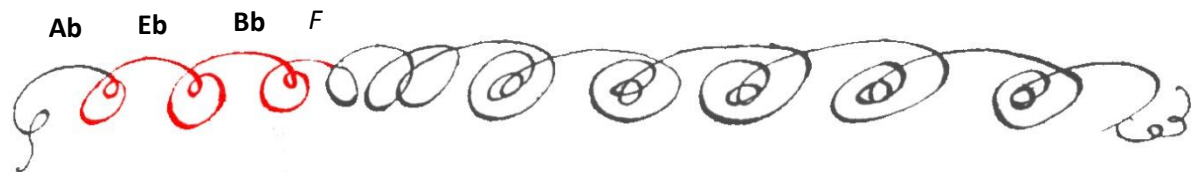
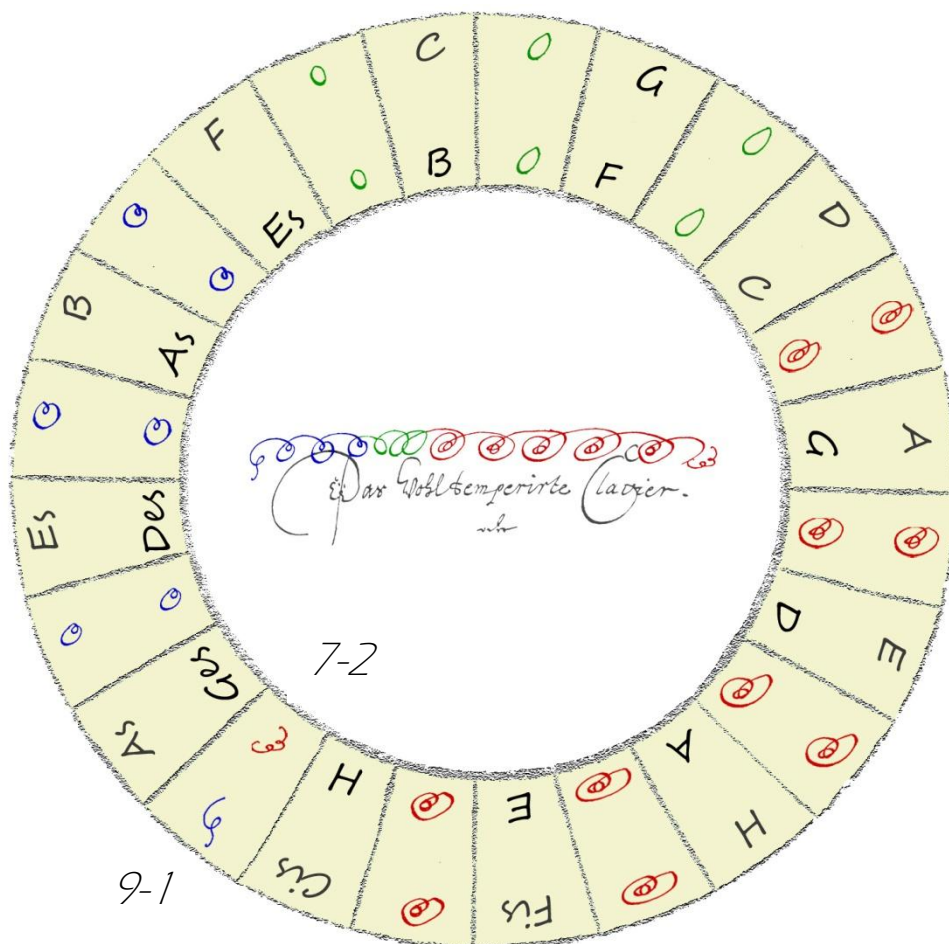The clockwise loops represent the tempering of fifths going towards the sharps.

Results

As described in [1] tempering is given by the following table:

Symbol	Number of small loops	Tempering
◯	0	0 Hz
◑	1	1 Hz
◎	2	2 Hz
⌡	1	1 Hz
⌇	2	2 Hz

This yields the Cammerton temperament 9-1, which is paired with its Cornet-ton transpose 7-2 [1].

Discussion

Consider the following:

Symbol	Interpretation
	Tuning for subdominant, tonic and dominant fifths of C
	Tuning for fifths towards the flats
	Tuning for fifths towards the sharps
	Enharmonic spellings of interval closing the circle of fifths

The above allows any arbitrary temperament to be deduced, for example:

	12-TET	¼ C Meantone	Pythagorean	-1/9 PC	-1/8 PC
	-1/12 PC	-1/4 SC	Pure	Pure	Pure
	-1/12 PC	-1/4 SC	Pure	-1/9 PC	-1/8 PC
	-1/12 PC	-1/4 SC	Pure	-1/9 PC	-1/8 PC
	-1/12 PC	Wolf	Wolf	-1/9 PC	Pure

However, by considering topological characteristics of the symbols, i.e. the number of small loops, the solution is constrained. Comma semantics are then p*recluded*, however, as the end points have inconsistent numberings (1 and 2, respectively).

As shown in an earlier paper there are two candidate solutions [1]. The current article has considered further topological aspects, namely the clockwise and anticlockwise orientations of certain loops, to derive a unique solution.

Reference

1. John Charles Francis, The Esoteric Keyboard Temperaments of J. S. Bach, EUNOMIOS Jour., 01 February 2005.